A MATHEMATICAL MODEL FOR OPTIMAL CORPORATE ALLIANCES: EVIDENCE FROM JAPAN

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ABSTRACT

In this paper, we are proposing a new mathematical model for choosing business partners in corporate alliances. We have used the real corporate data of 152 Japanese companies graded on eight characteristics. These characteristics include sales force, technical ability, capital resources, human resources, production capacity and other items that represent management resources. These characteristics can be described using a one-dimensional matrix. The subtraction of two such one-dimensional matrices results in a bipolar vector shows the relationship of the corporate alliance between two companies. The strength of a mutually complementary relationship is mathematically represented as the distance from the maximum point. The proposed model was implemented in the Python programming language. We have analyzed 152 Japanese companies and the computed results of the mutually complementary strength coefficient. Based on this, we have verified the functionality of the model. By using the proposed model, we can determine which candidate(s) from multiple potential companies form the best-suited alliance.

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KEYWORDS: Mathematical Model, Corporate Alliance, Mutually Complementary Relationship, Management Resources, Python Programming Language, Japanese Companies

INTRODUCTION

The first author of this paper consulted 152 Japanese companies between May 2008 and March 2015, particularly for the arrangement of corporate alliances for the purpose of business development and increasing sales. The first author is the president of a consulting firm that specializes in the arrangement of corporate alliances.

This paper defines a corporate alliance as an exchange of management resources, namely complementarity of strengths and weaknesses between two companies in order to create new business and increase sales revenue of their current businesses, regardless of the presence or absence of a binding contract or capital relationship, by continued cooperation, to share the results. These definitions are originally from Yoshino and Rangan (1995), which is the first comprehensive study on the different types, classes and definition of alliances.

Although most of the research on corporate alliances was mainly conducted after potential alliance partners had been found and selected, in this paper we will focus on the matching phase of a corporate alliance, which involves the finding and selection of potential alliance partners.

In prior research on corporate alliances, there has been no mathematical model to express the relationship of two companies in the establishment of a corporate alliance during the matching phase. Based on this, we propose a new mathematical model to express the mechanism of the formation of corporate alliances by expanding the RBV theoretical framework.

In this study, we consider a corporate alliance is successful when the mutually complementary relationship between two companies, which is when the balance of strengths and weaknesses of each company, is strong. In the construction of the mathematical model in this paper, we represent the strengths and weaknesses of a company with a one-dimensional matrix and bipolar vector. We mathematically define the strength of the relationship between two companies as the distance from the maximum point of a mutually complementary relationship. We have succeeded in constructing a mathematical model to calculate the mutually complementary strength coefficient as a numerical value. By using this model, we can select which company is the optimal alliance partner when there are multiple potential alliance partners when there are alternative options in the formation of a corporate alliance.

This proposed model has been implemented in the open source programming language Python. We have used the real empirical data of 152 consulted companies in Japan graded on eight different characteristics, mainly representing management resources, to evaluate successful and unsuccessful pairs of corporate alliances. This proposed model is flexible, so users can freely change the number of characteristics and grading method as well as experiment with different company data.

The proposed model in this paper can help to accelerate corporate alliance activity and encourage corporate alliances between companies in Japan as well as corporate alliances between companies in Japan and companies outside Japan. In this paper, we conducted research on companies within Japan, so we will also explain the current situation and trends of corporate alliances in Japan.

LITERATURE REVIEW

In prior research on corporate alliances, the theoretical background on the establishment of corporate alliances was prepared by Yasuda (2003, 2006, 2010) and Ushimaru (2007).

The establishment of corporate alliances relies primarily on the Resource-Based View (RBV), originally started by Wernerfelt (1984) and Barney (1991) and expanded to research of corporate alliances by Das and Teng (1998, 2000), Lavie (2006) and Yasuda (2003, 2006, 2010).

In this paper, we will use the RBV as it applies to the hypothetical establishment of a corporate alliance between two companies as it pertains to matters such as the necessary management resources for business deployment and the mutual complement to each company's strengths and weaknesses.In regards to the research on strategic management, Barney (1991) has presented a Resource-Based View (RBV), in which the firm resources for generating sustained competitive advantage is analyzed using the four empirical indicators of the potential of firm resources to generate sustained competitive advantage: value, rareness, imitability and substitutability.

Firstly, in the application of RBV for research on corporate alliances, Das and Teng (1998) considers the cooperative adjustment of financial, technological, physical and management resources and two types of risk: relational risk and performance risk.

Additionally, Das and Teng (2000) considers the systematic application of RBV to strategic alliances. The theory covers four major aspects of strategic alliances: rationale, formation, structural preferences and performance. In certain resource characteristics such as imperfect mobility, imitability and substitutability, there are two dimensions of resource similarity and utilization, resulting in the four types of alignment: supplementary, surplus, complementary and wasteful.

In this way, RBV has expanded to corporate alliance research. Moreover, Yasuda (2003, 2006, 2010) proposed the perspective of exchange of management resources as a new analytical approach for strategic alliances. Yasuda (2003, 2006, 2010) simplified the types and nature of firm resources down to five categories of management resources: 1. Technological resources, 2. Human resources, 3. Production resources, 4. Sales resources, 5. Financial resources. Yasuda (2003, 2006, 2010) concluded that corporate alliances are an exchange of management resources. This paper furthers this concept and framework, and we have applied this concept as a theoretical base for our research.

In the mathematical model proposed in this paper, the main concept has been printed in Japan in Tomita and Takefuji (2015). In this paper as well, using Tomita and Takefuji (2015) as a base, we expand upon the research in which a mathematical model was constructed with four characteristics. In this paper, we expand the number to eight characteristics and apply this to select the optimal corporate alliance partner from multiple potential partners.

In Lavie (2006), the RBV is extended in the research of network resources of interconnected firms to conclude that after reassessing the heterogeneity, imperfect mobility, imitability, and substitutability conditions, the nature of relationships may matter more than the nature of resources in networked environments. In our research as well, we put emphasis on the relationship itself between two companies.

One other theory introduced in corporate alliance research by Yasuda (2003, 2006, 2010) and Ushimaru (2007) is the Transaction Cost Theory. It proposes that in the event of lower costs relative to normal market transactions, it is better to form a corporate alliance with an external company rather than to internalize practices. However, even though there is mention of corporate alliances as a phenomenon of corporate behavior, this does not explain how to select business partners in a corporate alliance. For this reason, we do not use this theory in our research.

In Game Theory, alliance partners are already established since the theory analyzes the behavior and relationships of a fixed number of companies already in an alliance. As such, in this paper the author has determined this is not suitable as a theoretical background during the matching phase of an alliance when a company is searching for candidate partners.

Additionally, as in Das and Teng (2002), for companies in an alliance, there are exchanges of non-economic resources as per social exchange theory, but in this paper we are concerned with economic resource exchange since we are looking to study the economic benefits in an alliance, and not view this as merely a theoretical background.

In the data breakdown section of this paper, there were many small sized companies included, as the study of alliance strategies of small firms by Gomes-Casseres (1997), though no mathematical model used for that study was presented.

In Mitsuhashi and Greve (2009), although there was research conducted into an alliance matching model, the concepts and the research focused on ideological issues of partner companies within the shipping industry and no mathematical model was presented.

In terms of weighted analysis of corporate alliances by statistical methods, there have been many studies. Thus, although the study of quantitative analysis is present in the other studies on alliances, because there is no mathematical model or mechanism that represents whether the companies in an alliance are successful or not, I have not been able to numerically calculate any values to express the relationship between two companies in a corporate alliance.

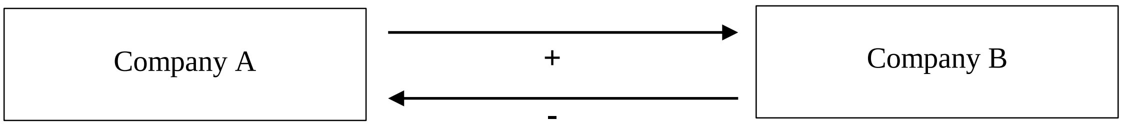
MATHEMATICAL MODEL DEVELOPMENT

Comprehending Mutual Complementarity and Application of the Physical Model

As we construct a mathematical model that shows the mechanism of the establishment of corporate alliances, we devise a theory based on the physical spin glass magnetic force model, which is the theory that the N and S poles attract each other.

In the idea of a mutually complementary alliance model, the strengths of company A will complement the weaknesses of company B, and vice versa. It then becomes fundamental for the strengths of company B to complement the weaknesses of company A. If the complements from one of the companies or both are small, the mutually complementary strength will also be small. Conversely, if the complements from one company or both are large, the mutually complementary strength will be large. That is, the mutually complementary relationship in a corporate alliance is a bipolar model based on the mutually attracting forces between two companies.

Figure 1: The bilateral appeal between two companies in corporate alliances



In a corporate alliance, it is fundamentally important that the bilateral appeal between two companies exists and is strong. The stronger the bilateral appeal is, the most likely an established corporate alliance will be successful.

Expressing the Strengths and Weaknesses of a Company as a One-Dimensional Matrix

The strengths and weaknesses of companies A and B can be expressed as a one-dimensional matrix of eight characteristics representing management resources, each graded with values between 1 and 5. The eight characteristics are mainly based on the Resource-Based View (RBV) as explained previously. Also, the integer values represent the score evaluating the strength and weakness of these characteristics for each company.

As an example, consider the two following companies,
Company A a = (1, 3, 4, 2, 5, 1, 3, 1) (1)
Company B b = (4, 1, 1, 3, 1, 5, 3, 1) (2)

From the above, the result "c" can be shown by subtracting the values of each of the characteristics of Company B from Company A in order to get a directional bipolar vector with values for each characteristic ranging from 0 to 4 (positive or negative). That is to say, we can express the mutually complementary relationship between two companies as a bipolar vector.

Company A – Company B
c = a-b = (-3, 2, 3, -1, 4, -4, 0, 0) (3)

Note that in this particular case, when calculated using the programming model described later, the mutually complementary strength is 11.997 and the related coefficient is 0.530 by making the calculation as explained in the section of "Mutually complementary strength and the related coefficient".

How to Determine the Maximum Mutually Complimentary Relationship as a value

The strengths of the mutually complimentary distance are expressed by measuring the distance from the largest mutually complimentary point of strength.

Namely, in regards to the bipolar vector of the length from 0 to 4 of the eight characteristics, the maximum mutually complementary value determined from taking two sets of half the number of characteristics (4) with a maximum length of 4 for each, which is the longest possible bipolar vector bilaterally.

(8 characteristics / 2) * Maximum length of 4 = (16,-16) (4)

The distance (d) between two points is calculated as follows:

$$d = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} \quad (5)$$

The maximum value of the mutually complementary strength of (16,-16) is shown as the distance from (0, -0) to (16, -16), which becomes

$$\sqrt{(16-0)^2 + (-16+0)^2} = 22.63 \quad (6)$$

The mutually complementary strength is a value between 0 and 11.3, with a large value representing a large mutually complementary strength.

When the distance from the maximum value of (16,-16) is small, it indicates that the mutually complementary strength is strong. Since it is simpler to subtract from larger numbers, we have inverted the magnitudes of the values.

Summation of Positive and Negative Integers

As an example, in companies A and B mentioned previously, the value expressing the provided strengths from company A to company B is the summation of positive integers (plus' bipolar vector):

$$2 + 3 + 4 = 9 \quad (7)$$

This number (9) shows the strengths of company A that complement the weaknesses of company B.

Conversely, taking the summation of negative integers (minus' bipolar vector):

$$-3 + (-1) + (-4) = -8 \quad (8)$$

This number (-8) shows the strengths of company B that complement the weaknesses of company A.

The two numbers (9, -8) show the mutually complementary relationship between company A and company B.

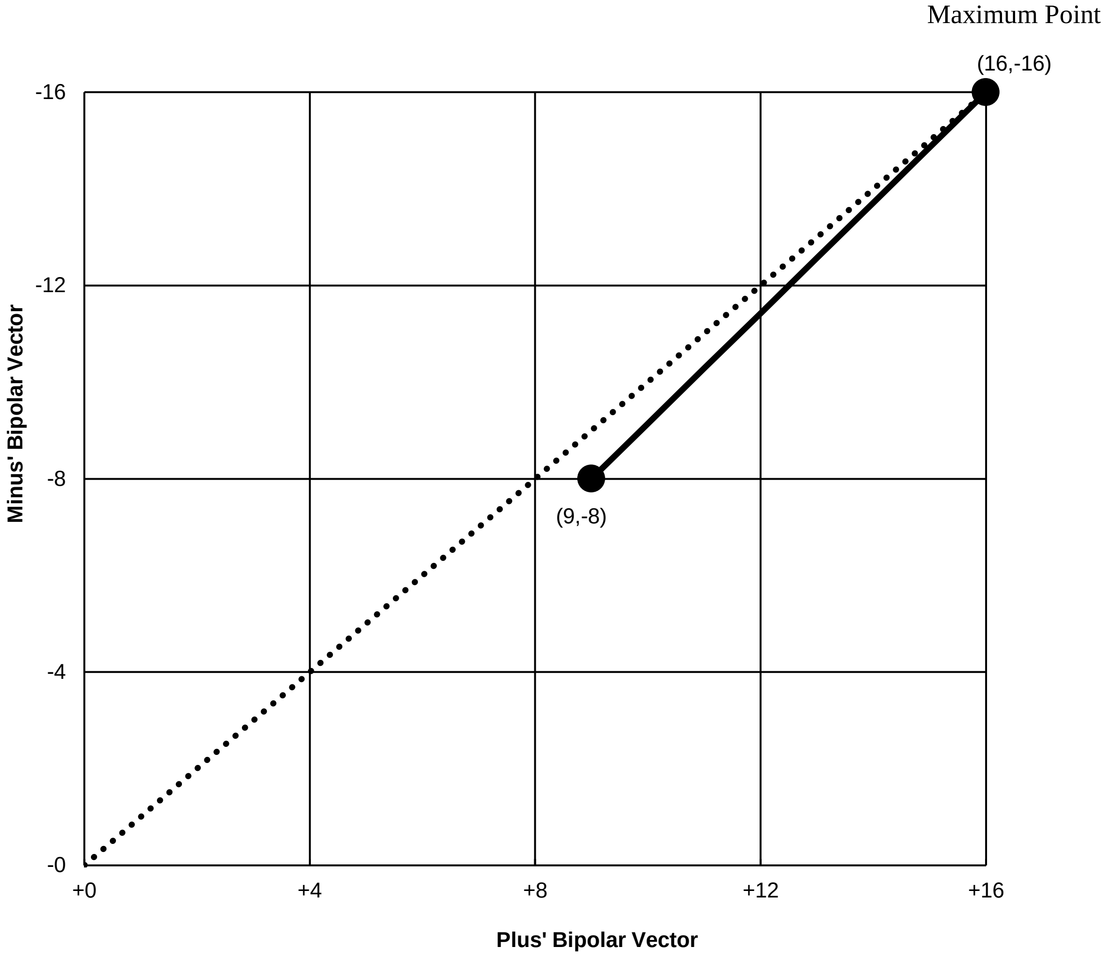
The Strength Expressed as the Distance from the Maximum Point

Based on the construction of this mathematical model, we express the mutually complimentary strength by measuring the distance from the strongest mutually complimentary point, namely the maximum point.

For example, with a mutually complementary strength of (9,-8) for companies A and B, it is possible to calculate the distance from (16,-16) by means of subtraction from the maximum value.

$$\sqrt{(16-0)^2 + (-16+0)^2} - \sqrt{(16-9)^2 + (-16-(-8))^2} = 11.99 \quad (9)$$

Figure 2: The Bipolar model of mutual complementarity for alliances



This figure shows the example representing the mutually complementary strength of 2 companies by the distance from the maximum value of the mutually complementary strength. In case of 8 characteristics, the maximum value of the mutually complementary strength is (16,-16) and the point presenting the mutually complementary strength of Company A and Company B is (9,-8).

<u>Mutually complementary strength and the related coefficient</u>

Up until this point, the eight characteristics have been described as a mutually complementary bipolar vector. Here is the general equation used to express this.

The mutually complementary strength derived and explained above, can be expressed by the following formula:

$$\sqrt{2 \times \left(\frac{4 \times len(c)}{2}\right)^2} - \sqrt{\left(\frac{4 \times len(c)}{2} - plus\right)^2 + \left(\frac{-4 \times len(c)}{2} - minus\right)^2} \quad (10)$$

In the above formula, len(c) is the number of characteristics, plus=Σ (positive integers), and minus=Σ (negative integers).

When this value is normalized to a value between zero and one it becomes easier to handle. The relative mutually complementary strength can be calculated from the following formula, and is here forth defined as the mutually complementary strength coefficient.

$$1-\frac{\sqrt{\left(\frac{4\times len(c)}{2}-plus\right)^2+\left(\frac{-4\times len(c)}{2}-minus\right)^2}}{\sqrt{2\times\left(\frac{4\times len(c)}{2}\right)^2}} \quad (11)$$

In the above formula, len(c) is the number of characteristics, plus=Σ (positive integers), and minus=Σ (negative integers).

Limitations of this Mathematical Model

We have already noticed that the model has several limitations, as described below.

In the model, the maximum point is calculated from the maximum length of half the number of characteristics. In the event the number of characteristics is an odd number, of course we are unable to divide by 2, so for that reason we must introduce a dummy characteristic.

In another way, if the number of characteristics is odd, in order to apply the model, we understand there will actually be two maximum points. In this case, despite the fact the point does not actually exist, we calculate the midpoint of the line connecting the two maximum points on the assumption this is a linear equation and consider this the maximum point. Although this is not described in detail in this paper, this is considered content for a future paper.

Regarding the seven types of corporate alliances shown in Tomita (2014), corporate alliances between companies in different business areas have not been included in the proposed model in this paper, because this type of a corporate alliance is a calculation of addition, it is not complementary. Since this is an "addition model" which is, we plan to add this to the mutually complementary model. Also, regarding alliance formation, motivation from each of the companies is an important factor, but that has not been included in this model. Since this is a "synergistic model", in the same way it is planned for addition to the mutually complementary model. These will be content for a future paper, for which the contents have already been submitted in Japanese in a separate academic paper.

DATA AND METHODOLOGY

How the Data of 152 Companies was Constructed

The author of this work consulted 152 Japanese companies between May 2008 and March 2015 regarding new business development and increasing sales. In the enterprise data used in this study acquired from the author over a span of seven years, one feature is that it contains some general private, informal information that has not been published by these companies.

The data used in this paper was collected from the materials submitted by the consulting companies and advice obtained in meetings (with managers and employees). Information obtained from partners of referral agreements was also incorporated into this data. Rather than a superficial examination of data from surveys, the authors themselves become a participant in the analysis of business activities.

It should be noted that in previous studies of a number of alliances, research was conducted on major companies such as those listed on the stock market. For example, Doz and Hamel (1998) and Yasuda (2003, 2006, 2010) target large companies, but one feature of this paper is that small and medium-sized companies are also of interest.

Table 1: Breakdown of the data from 152 companies.

Location	**Tokyo (all 23 wards)**	**Metropolitan**	**Non-metropolitan**	
	71%	13%	16%	
Size	**Listed (Inc. subsidiaries)**	**Small-mid sized with long longevity**	**Early Stage Startup**	
	13%	40%	47%	
Industry	**In-house Manufacturing**	**Contract Manufacturing**	**Sales/Marketing**	**Pro/Consulting**
	17%	35%	30%	11%
IT/Non-IT-Related	**IT Related**	**Non-IT Related**		
	40%	60%		
Sales	**< 1 Billion Yen**	**Between 1~10 Billion Yen**	**Over 10 Billion Yen**	
	78%	15%	7%	
Employee Size	**< 20**	**20~100**	**> 100**	
	56%	32%	13%	

This Table shows the data from 152 consulted companies in Japan from 05/2008 to 03/2015. Data is broken down into region, size, industry, IT/Non-IT relation, sales and number of employees.

Grading of the Eight Characteristics

In regards to the strengths and weaknesses, the eight characteristics used in this study are: 1. Sales Force, 2. Technical Ability, 3. Creativity of Ideas, 4. Capital Resources, 5. Human Resources, 6. Production Capacity, 7. Branding and Reliability, 8. Flexibility of Organization. Note that for the 152 consulted companies, the author graded each of the eight characteristics from 1 to 5, with 5 being the largest value. In order to address the issue of subjectivity when grading the characteristics, each characteristic was comprised of four criteria in the grading process. Additionally, another consultant graded the characteristics using the same method.

In business studies, in order to ensure the validity of the research results, the accuracy of the materials and data is of the utmost importance, so objectivity is required. For the data used in this study from 152 consulted companies graded on eight characteristics, we become the focus of the grading of the strengths and weaknesses of each company. In other words, the problem of subjectivity with those conducting the evaluation remains. In order to eliminate the subjectivity of the evaluator, we have taken the following two measures.

In the evaluation process for the eight characteristics, these characteristics are further divided into four factors for each, and based on the summary of the results of these four factors, the value of the characteristic is evaluated as a final step.

For example, when evaluating the sales force of each company, there are factors to consider such as if the number of salesmen is large and if the salesman are active. For capital resources, there are criteria to determine the state of the financial figures. For more information on the exact factors used in evaluating each of the eight characteristics, please refer to Appendix A. Regarding the eight graded characteristics of strengths and weaknesses, each characteristic has four set factors. The grading is performed based on these factors.

Grading was conducted by not only one person, but rather another consultant as well, with the averaged results of both used to determine the final grade. In this manner, the usage of a second consultant to grade the eight characteristics of strengths and weaknesses of each company from at least two consultations as well as the grading of four factors for each characteristic to determine the final grade, we have reduced the subjectivity of the study as much as possible.

However, note that the purpose of this paper is to propose a mathematical model for alliances. The data sample and grading methods used here are but a single application of this model and should not be considered representative of all typical data from Japan.

Since the mutually complementary strength and its coefficients are calculated based on the grading of each characteristic, the graded values may change depending on the rules for grading, so the mutually complementary strength and the coefficient will also be affected. Future grading system changes will be needed to improve accuracy.

The purpose and significance of this study is to express and evaluate the matching phase of an alliance between companies mathematically, and to implement the model in an open source programming language. In order to achieve this, we verified the functionality of the model with a sample of 152 consulted companies. However, ultimately this is only one data sample.

Consulted Companies and Outside Consulted Companies

From the almost 4,000 potential alliance partners with which the first author has relations, even though there exist companies outside of the introduced 152 companies, there are numerous other companies that cannot be sufficiently evaluated based on the eight characteristics. In the introduction of the consulted companies by the author, there exist companies outside of those consulted that were matched and have established a successful alliance. Because the available information on some companies is incomplete, we cannot accurately apply the proposed model without distorting the data, and thus and we limit our study to the 152 consulted companies as to whether or not an alliance was successful or unsuccessful.

In this data set of the empirical data from the consulted companies, the study was conducted between May 2008 and March 2015, and the companies had been consulted at the time of the survey from a minimum of two months to a maximum of three years. During the consultation, which is to say the period of arranging alliances, there may be small gaps, but in the evaluation, all consulted companies have been graded.

Tallying the Successful and Unsuccessful Alliances

In the data of the 152 consulted companies, there were 121 successful alliance combinations and 30 unsuccessful ones. Additionally, we also examine all possible alliance combinations from within the 152 consulted companies, a maximum possible total of 11,476 combinations (152 x 152 - 152).

Table 2: Number of Successful and Unsuccessful Alliance Pairs

Category	**Number**
Successful Alliance Pairs	121
Unsuccessful Alliance Pairs	20
Total Number of Possible Pairs	11,476 (152 x 152 - 152)

In the above table, from the 152 consulted companies, we can see the number of alliances that were successful, unsuccessful, and the total number of possible pairs.

This paper proposes a mathematical model to express the mechanism during the matching phase of corporate alliances. For this reason, in verification of this research, we define a successful alliance as, after the introduction of two companies to each other, forward progress in the development of new business or expansion of their businesses. Conversely, we define an unsuccessful alliance as, after the introduction of two companies to each other, no forward progress for either company in any business development or expansion of their businesses.

Also, in Japan the establishment and promotion of alliances is not necessarily limited to contracts. Rather than a verbal agreement, some alliances are formed at the time when agreements and invoices are sent, at which time referral fees or margin payments are incurred. Since Japanese business culture considers such methods for forming alliances, it is not a requirement for companies to have a formal contract when forming an alliance.

Demonstration and Analysis using the Python Language

The calculation of the mutually complementary strength coefficient was executed from a script written in the open-source programming language "Python".

Although the proposed model did not have to be implemented specifically in Python, as an open source programming language Python is convenient, allows a high degree of freedom, and has been increasingly popular in recent years. For these reasons we have decided on the Python language as an implementation tool in this research. With the freedom of this open source language, we have constructed a mathematical model through trial and error and proposed the mathematical model as a flexible model. Recently in business administration academia, although it has been common to make use of statistics software such as Stata, interest in the Python programming language is growing. We can say this paper is one of the pioneers of this research.

RESULTS

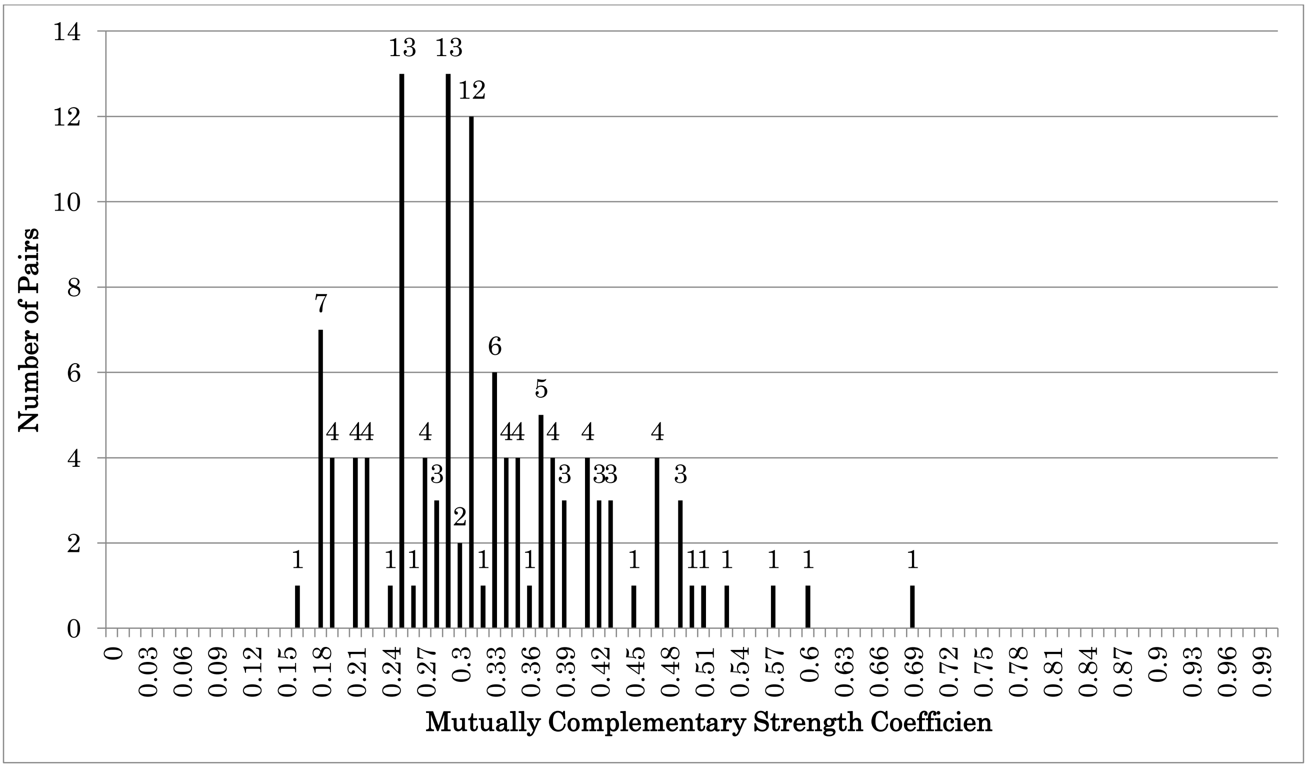
Distribution of the Mutually Complementary Strength Coefficients for Successful, Unsuccessful and All Alliances

In the successful alliances (121 pairs) shown in Figure 3, the distribution of the mutually complementary strength coefficients is shown in increments of 0.05. Likewise, in the unsuccessful alliances (30 pairs) shown in Figure 4, the distribution of the mutually complementary strength coefficient is shown in increments of 0.05.

In addition, in the successful, unsuccessful alliances, and total number of possible pairs (11,476 pairs) referred to as "All Possible", the distribution of the mutually complementary strength coefficient is also 0.05 (though not explicitly indicated) as shown in Figure 5.

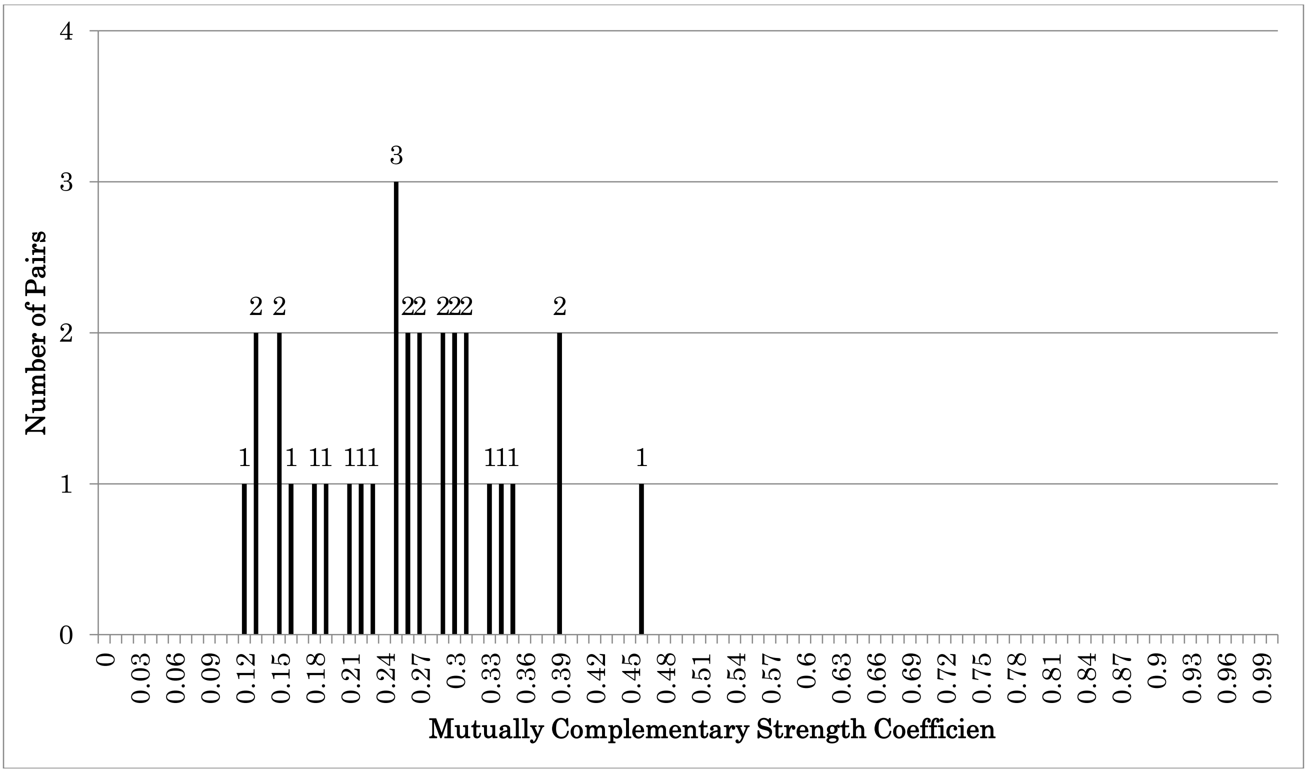
These superimposed results of the distribution of mutually complementary strength coefficients for all possible combinations of alliances between the total number of possible pairs are shown in Figure 5. In the distribution over the combination of all alliance combination possibilities, the shape closely resembles a normal distribution shape, which allows us to estimate an average of the mutually complementary strength coefficients. From all combinations of alliances, the coefficient for successful alliances can be seen in the larger values, while for unsuccessful alliances, there is a trend towards the lower coefficient values.

Figure 3: Distribution Graph of the Mutual Complementary Coefficient for Successful Alliances



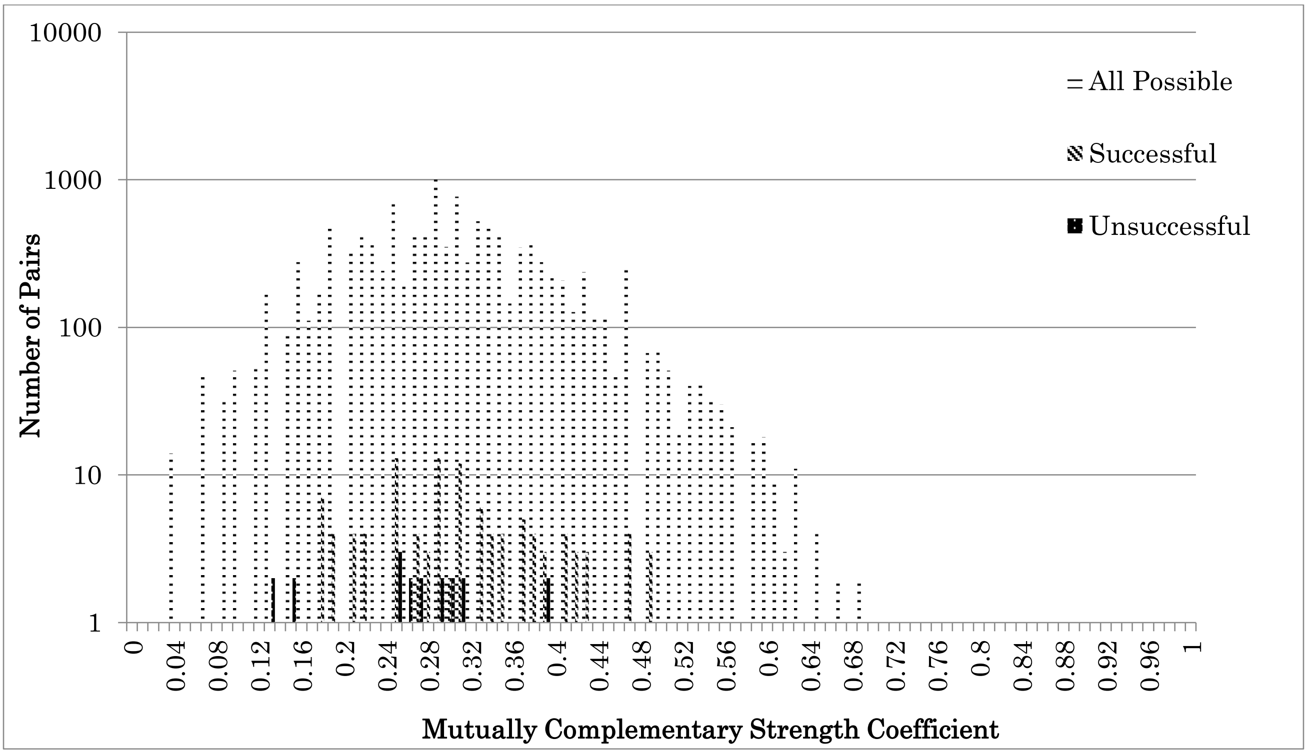
This Figure shows the distribution of the mutually complementary strength coefficients for successful corporate alliances.

Figure 4: Distribution Graph of the Mutual Complementary Coefficient for Unsuccessful Alliances



This Figure shows the distribution of the mutually complementary strength coefficients for unsuccessful corporate alliances.

Figure 5: Distribution Graph of the Mutual Complementary Coefficient for all Possible Alliance Combinations



This Figure shows the distribution of the mutually complementary strength coefficients for all possible corporate alliances.

In the distribution graphs of the mutually complementary strength coefficients for successful and unsuccessful alliances (Figures 3, 4), even when an alliance is unsuccessful, it has a large mutually complementary strength coefficient. When the mutually complementary relationship is strong, which is to say, when the mutually complementary strength coefficient is large, the hypothesis that the alliance is likely to be successful still holds.

The Average of Mutually Complementary Strengths Coefficient for Successful, Unsuccessful and All Alliances

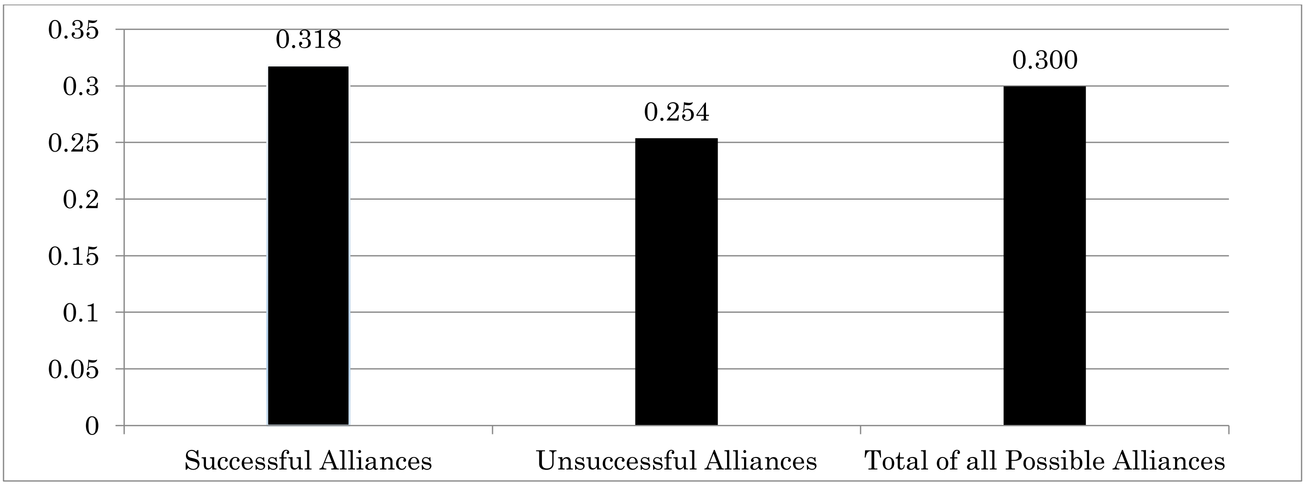
As a result of calculating the mutually complementary strength coefficient programmatically, for 121 pairs of successfully allied companies, the average coefficient was 0.318, whereas for 30 company pairs for which the corporate alliance was not successful, the average coefficient was 0.238. This indicates that the complimentary strength coefficient is higher for two companies in a corporate alliance.

Regarding why the coefficient value does not differ much for companies in a corporate alliance versus those not in one, we begin by stating that we think there is a mutual complement between two companies, which is why they are drawn together.

It should be noted that the average coefficient overall for the 152 company alliances was 0.300. Figure 2 shows the result of tallying the coefficients for all 152 company alliances. Hence, this mathematical model of the bipolar mutual complement constructed from the actual company data confirms the function as valid. Take note that values have been rounded to the nearest thousandth.

As for the results, in Figure 3, there are 121 cases of successful alliances, with an average mutually complementary strength coefficient of 0.318. There are 30 cases of unsuccessful alliances, with an average mutually complementary strength coefficient of 0.254. As you can see in the figure, the average mutually complementary strength coefficient for a successful alliance between two companies is higher than the average over all possible company pairs.

Figure 6: Average of Mutually Complementary Strength Coefficient for Successful, Unsuccessful and Total Possible Alliances



In this chart, we can see the average of mutually complementary strength coefficients for successful and unsuccessful alliances as well as for all possible pairings of the 152 consulted companies. This is also illustrated in the accompanying graph. Values were rounded to the nearest thousandth.

As we can see from the distribution in the above graph, the coefficient for successful alliances is larger than the value for all possible alliance combinations. Similarly, the average value for unsuccessful alliances is a smaller value than that for all possible alliance combinations. Therefore, we have shown that successful alliances will have a larger coefficient value which indicates that the mutual complementary relationship is strong.

In this manner, the mutually complementary strength coefficient was calculated using the proposed mathematical model in this paper by a Python script and the model was verified to function in practice.

Application of this Model for Selection of an Optimal Partner from Multiple Potential Partners

Making use of this model, let us find the best-suited alliance partner in the case of multiple potential alliance partners. Using the results from the Python script we wrote to implement this model, let us apply what we have proposed.

Using the proposed model, we can determine which pairs of companies will best form a corporate alliance based on which pair has a stronger mutually complementary relationship when there are multiple possible combinations of companies for a corporate alliance.

By calculating the mutually complementary strengths and related coefficients for all pairs, it becomes possible to select the optimal corporate alliance.

For example, consider the simple case of the following three companies, (I, J, K) for which they are attributed values from 1 to 5 for each of their 8 characteristics as represented in the following one-dimensional matrices.

$i = (1, 3, 4, 2, 5, 1, 3, 1)$
$j = (4, 1, 1, 3, 1, 5, 3, 1)$
$k = (3, 5, 2, 4, 2, 3, 5, 4)$

From these three companies, by subtracting the one-dimensional matrices associated with each possible pair of companies, we can determine the mutually complementary relationship between companies I and J, I and K, and J and K. These relationships are represented here with the following equations.

$$x=i-j \tag{12}$$
$$y=i-k \tag{13}$$
$$z=j-k \tag{14}$$

Based on the results from our Python script, the mutually complementary strength and the related coefficients for x, y, and z are shown below:

Strength of x (=i-j) =11.99
Strength Coefficient of x (=i-j) =0.530

Strength of y (=i-k) =11.22
Strength Coefficient of y (=i-k) =0.496

Strength of z (=j-k) =9.025
Strength Coefficient of z (=j-k) =0.398

According to these values, when company I has to choose between companies J and K, we can see “x” is larger than “y”, therefore company J is determined to be the most suitable partner for company I.

$x > y > z$

In the same way, when company J has to choose between companies I and K, we can see that “x” is larger than “z” when we compare them, so from the viewpoint of company J, company I is the most suitable partner.

As well, when company K has to choose between companies I and J, we can see that “y” is larger than “z”, so company I becomes the most suitable partner for company K.

Furthermore, when determining the optimal selection from these three companies, we see that “x” is larger than “y” and “y” is larger than “z”. Therefore, from companies I, J and K, The pair of company I and company J is the best-suited pairing from among these three potential partners.

By extension of the methods in this proposed model, from a number “m” of companies, we can determine the best-suited partner from a number “n” of potential alliance partners, which is to say, a total of $\binom{m}{n}$ possible alliance pairings.

The Current Situation and trends of Corporate Alliances in Japan

In this paper, by making use of the empirical data from 152 consulted companies in Japan, we have calculated the mutually complementary strength coefficient using the proposed mathematical model in Python. Although this data cannot be said to be representative of all typical Japanese corporate data, the analysis was conducted on corporate alliances in Japan.

Since the subtitle to this paper is “Evidence from Japan”, I would like to explain the situation of corporate alliances in Japan. As mentioned in Hamel, Doz, and Prahalad (1989) based on corporate alliances between Japanese and Western companies in the 1980s, even though Japanese companies were actively seeking alliances with Western companies for acquisition of skills and technology, basically Japanese companies are not particularly accustomed to alliances, and tend to be an inward-thinking culture and closed innovation, which is the limited focus of research and development to only inside the company, also known as NIH syndrome (Not Invented Here Syndrome).

As noted in Yasuda (2003, 2006, 2010), when looking at the “alliance matrix”, which divides corporate alliances into four types through two axes, of which the axes represent (1) Alliances in the same industry or different industries (2) Exchange of the same or different management resources. We can see that in Japanese companies, corporate alliances are formed in order to avoid excessive competition and to make their business more effective, so corporate alliances in the same industry and of the same management resources were the most numerous cases. For example, in the semiconductor industry, Hitachi Corporation, Mitsubishi Electric, and NEC Corporation formed a strategic alliance, and as a joint venture (JV), operated as Renesas Electronics Corporation, and is the largest typical case. Additionally, even though it is not a corporate alliance and has only progressed to mergers, in the steel industry there is Nippon Steel & Sumitomo Metal Corporation (Integration of Nippon Steel and Sumitomo Metal), and in the trust banking industry there is Sumitomo Mitsui Trust Bank, Limited (Merger of Sumitomo Trust & Banking and Mitsui Chuo Trust). In these examples, these are companies in the same industry, exchanging the same management resources. In this way, we can see that with past corporate alliances in Japan, alliances in the same industry of the same management resources are the most common.

However, in the current situation of corporate alliances in Japan, seeing as the Japanese market tends to shrink as it matures, companies began to more actively seek corporate alliances in order to acquire new revenue.

On the first trend of the current situation of corporate alliances in Japan, there are corporate alliances not only in the same industry, but a growing number of corporate alliances have been established between companies in different industries or exchange different management resources as well.

To list a few examples, Gakken Holdings Co., Ltd., a leading enterprise of kindergarten and elementary school education made a corporate alliance with Kawai Musical Instruments Mfg. Co., Ltd., the second largest piano manufacturer in Japan to create a new business development for children to expand their existing business. The second largest chemical wrapping sheet manufacturer Kureha Corporation also made a corporate alliance with Toppan Printing Company Ltd., one of the largest printing companies in Japan, to develop new kitchen supplies. In the power industry, Kansai Electric made a corporate alliance with the telecommunications company KDDI Corporation for the sale of electric power, cellular phones and optical fiber lines in a single package in accordance with the deregulation of the Japanese government's rules on electric power. Finally, one of the largest supermarket chains Inageya Co., Ltd. made a corporate alliance with the low cost apparel store chain Shimamura Co., Ltd. to cooperatively open new stores since they share the same consumer target.

Next, on the second trend of the current situation of corporate alliances in Japan, the number of alliances between large companies and start-up companies is rapidly increasing. For example, the startup company Qrio, Inc. in the field of IoT (Internet of Things) received investment from Sony Corporation and launched a corporate alliance with Sony, which within a year completed the development of new products and then acquired new sales revenue. In Fintech's field (the combination of finance and technology), the start-up Money Forward which produced a personal accounting book as a mobile app received an investment from one of the largest credit card companies Credit Saison Co., Ltd. which in turn became a corporate alliance in order to promote their business development. In the field of artificial intelligence, auto manufacturer Toyota Motor Corporation invested in the startup Preferred Network, Inc., innovator of new algorithms of artificial intelligence, which in turn formed a corporate alliance for the purpose of developing automatic operation technology. These are some of the cases of corporate alliances between large companies and startups classified as activities of open innovation, noted by Chesbrough (2003) as viewed from the perspective of the larger company, namely companies listed in the first section of the Tokyo Stock Exchange.

Furthermore, the third trend of the current situation of corporate alliances in Japan includes international alliances (corporate alliances between companies in Japan and companies outside Japan), which is not limited only to between manufacturers for product development, but also other fields, especially internet related services. There is an increase in the number of companies outside Japan seeking corporate alliances with Japanese companies to expand into the Japanese market. For example, in order for the movie and television drama distributor Netflix, Inc., listed on US Nasdaq, to expand into Japan, they made a corporate alliance with Softbank Corp. (one of the largest telecommunications companies in Japan), Fuji Television Network, Inc. (Major television network), and Bic Camera Inc. (one of the largest home electronics retail chains). From this corporate alliance, Netflix has taken the strategy of making corporate alliances with companies that provide the greatest chance of success, and from the perspective of the Japanese companies, this creates a new business opportunity. Apart from this, the high quality Internet Service Provider Internap Corporation, listed on US Nasdaq, in order to expand into Japan with a high expectancy of success, they made a corporate alliance with Japan's largest telecommunication company NTT (Nippon Telegraph and Telephone Corporation) Group, and as a Joint Venture, formed Internap Japan Co., Ltd., which is currently operating in Japan. In the future, the number of these types of international alliances is expected to increase.

Continuing to the fourth trend of the current situation of corporate alliances in Japan, In America, corporate alliances between large companies are conducted and researched, but in Japan, even though there are cases of this, the corporate alliances between SMEs (Small and Mid-size Enterprises) appear to be a rapidly increasing type of corporate alliance. As for why this is so, there is a tendency of lack of management resources in SMEs. For this reason, it is important for these SMEs to form a corporate alliance to compensate for the strengths and weaknesses of the company in order to create new business and expand their business. Without using separate examples, by making use of the empirical data of 152 consulted companies in this paper we can express this trend. Of the 152 consulted companies, 40% of the companies are small to mid-sized with long longevity and

47% are startup companies, indicating that 87% of the data are SMEs. For this reason, even in the 121 pairs of successful corporate alliances, there are cases of one or both of the companies being SMEs included.

Lastly, as for the fifth trend of the current situation of corporate alliances in Japan, even in Japan, the number of businesses exchange meetings and business matching has been increasing. For example, there are regular gatherings and conferences for business matching. Some examples of this are Friend Link business meeting operated by Axel Media Co., Ltd., First Village business meeting managed by First Village Corporation, and Venture Alliance FES, operated by DYM Co., Ltd. Local governments and the Chamber of Commerce also hold various types of business meetings in order to stimulate business matching. There are also business matching and corporate alliance arrangement companies such as TC Consulting Co., Ltd., Ikashiaitai Co., Ltd., and NineSigma Japan, Inc. This shows an ever growing interest and need to promote corporate alliances with external companies.

In this way, we have shown that business matching and corporate alliances are active in Japan currently, as stated above with the four trends of corporate alliances.

Using the mathematical model proposed in this paper to calculate the values of the mutually complementary relationship between two companies in a corporate alliance, and we can accelerate corporate alliance activity in Japan. The reason why is that if we use the model, we can determine which company is the best suitable alliance partner. In this same way, the author believes that we can apply this model to companies outside of Japan in order to promote international corporate alliances between companies in Japan and companies outside Japan.

CONCLUDING COMMENTS

In this paper, we have proposed a mathematical model representing the mechanism behind the establishment of corporate alliances between two companies. By using this proposed model, we can calculate mutually complementary relationship between companies in a corporate alliance as a value.

In the proposed model we have mathematically expressed the mutually complementary relationship between two companies as a one-dimensional matrix, bipolar vector, and the distance from the maximum point. Based on this, we can calculate the value of the relationship between two companies in a corporate alliance. We named this value the mutually complementary strength coefficient. Thus, it has been made possible to mathematically capture and express the mutually complementary relationship between two companies, and is possible to determine the complementary strength coefficient.

This mathematical model was implemented in the open source programming language Python and confirmed to function based on the actual empirical data from 152 companies in Japan. From the grading of the eight characteristics representing management resources with values ranging from 1 to 5, the mutually complementary strength coefficient was calculated by a Python script that confirmed the functionality of the model.

Furthermore, when there are multiple possible candidates as an alliance partner, we can choose the best suited alliance partner by the comparison of the mutually complementary strength coefficient of each pair using the mathematical model proposed in this paper. That is to say, among three corporate candidates, we can use this mathematical model to calculate the coefficient to determine if a corporate alliance between two companies is best mutually for both companies when there are alternative alliance options.

The proposed model in this paper is a flexible model, so the number of parameters, weighting and the way of grading can be freely modified by users to apply the model to other various corporate data. We have released the Python script to implement the model on the Internet. I hope that many users freely adjust the number and types of parameters to conduct tests that produce results that more accurately represents actual real world situations using various corporate data. As such, this proposed model is considered a base for further development.

Currently in Japan, the importance and needs of corporate alliances for new business development and increasing sales has become greater. The trends and types of corporate alliance have diversified to become more cross-industry. There is also an increasing trend for corporate alliances between large companies and startups or between small to mid-sized enterprises. (SMEs). Also, there is a trend of an increasingly active number of business matching gatherings and consulting firms specialized in corporate alliances and business matching to accelerate activity of corporate alliances in Japan. This proposed mathematical model for corporate alliances can be used to stimulate such trends to promote activity of corporate alliances in Japan as well as encourage corporate alliances between companies in Japan and companies outside Japan.

Our research deals with the problems behind corporate alliances between companies. However, this proposed model is also applicable to alliances between people, namely human relationships if there is an importance for mutual complementarity in human relationships. For example, an applicable case in human relationships is how to establish business teams or to make marriage pairings. Additionally, we can even expand the usage of this model to alliances between countries in the past, present or future.

APPENDIX

The Four Factors for each of the Eight Characteristics of Strengths and Weaknesses (graded out of 5) used for Grading

Regarding the eight graded characteristics of strengths and weaknesses, each characteristic has four set factors. The grading is performed based on these factors. For the factors requiring a numerical evaluation, numerical criteria was provided.

1. Sales Force	**2. Technical Ability**
1. Salesman is active 2. Can perform sales management 3. Number of salesmen is large/small 4. The president has sales force capability	1. Possesses a unique technology 2. High quality technicians 3. Have spent a long time in the field 4. Research and Development sectors are substantial
3. Creativity of Ideas	**4. Capital Resources**
1. Environment to freely develop new ideas 2. Many/few people with good ideas 3. Constant development of business, ideas, or tech. 4. Presence of mechanism for new ideas	1. Has plenty of funds available 2. High capital adequacy ratio 3. Presence of a fund-raising force 4. High capital efficiency and profit margins
5. Human Resources	**6. Production Capacity**
1. Number of employees is great 2. Sense of surplus of employees 3. High recruiting force 4. Has a license for dispatch workers	1. Large factory capacity (machinery, space) 2. Have a lot of factory workers (blue collar) 3. Know-how of production management and quality control 4. Does not operate on a fabless policy
7. Branding and Reliability	**8. Flexibility of Organization**
1. Is a listed company or listed company subsidiary 2. Long industry history 3. Large sales 4. Focused on branding	1. Is corporate culture to tackle new tasks 2. Ideas are not just for discussion; put into practice 3. Respect for youth opinions, low degree of sectionalism 4. Corresponds to not only own, but external and foreign principles

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